

Intonation is a lifelong challenge for string players – which is not surprising, since perfect intonation is mathematically impossible. A note is in tune only in

relation to other notes, specifically their pitches. Pitch and tone colour are the subjective perceptions of the objective phenomenon of frequency. Frequency relationships are simply numerical ratios, and this fact allows us to think about them mathematically.

Perfect intonation is mathematically impossible because the relationships among notes depend on how you get from one note to the next. By way of demonstration, try this:

1. Tune your open strings until they're beatless (see box on page 60).
2. Play a beatless, pure major 6th, open G to E (figure 1). The E in this pure, beatless major 6th will be considerably lower than the E one whole step above open D.
3. Now play this E against the open A (figure 1). It will be very flat.

You could call the distance between D and E tuned to the open A a 'major whole tone', and the distance between D and E tuned to the open G a 'minor whole tone'.

The greatest conflict in intonation is the difference between intervals played sequentially and intervals played simultaneously. For example, an F sharp played as a pure double-stop with D is a much lower note than an F sharp leading note resolving on to G.

Understanding the theory behind intonation relieves players of the burden of trying to reconcile irreconcilables. The Andante of the Bach Sonata in A minor for solo violin provides a great example of this. It starts with a double-stop C and E, followed by a C and F (see figure 2). If these double-stops are tuned simultaneously, the distance between the E and the F will be much wider than the distance you would play when thinking of the E as a leading note to F. This raises a question of interpretation and taste.

FIGURE 1



FIGURE 2 Andante, Bach Violin Sonata in A minor



You may decide to favour the harmonic interval, and allow the sequential distance between the E and the F to be rather large for a leading note. Or you may play a wide (out-of-tune) major 3rd between the C and the E to bring the E leading note closer to F. Alternatively, you might compromise (which is what a piano does) by playing the E slightly high.

If you don't understand that a choice must be made, and try to tune a pure major 3rd and 4th as well as play the E as a leading note to the F, you will go crazy – or give up and stop listening so closely, perhaps burying the problem with a wide vibrato. >

All string players strive for perfect intonation, but is it really possible?
CHRISTOPHER BROOKS delves into the world of overtones, frequencies and beats in search of answers

MAKING WAVES

ISTOCKPHOTO

HARMONIC INTONATION VERSUS MELODIC INTONATION

Playing several notes simultaneously causes a physical interaction among their overtones, which is perceived as harmonic intonation (also known as just intonation). This physical interaction is most noticeable in consonant double-stops.

Sequential or melodic intonation is based on pitch memory and is thus more subjective. When notes follow each other, the salient relationship is the leading half step, usually upward but sometimes downward.

The strongest conflict between these two ways of hearing note relationships occurs with major and minor 3rds. How you approach this conflict is a question of taste and style.

When playing sequentially, either solo or with piano accompaniment, you should usually play a major 3rd as a major whole tone followed by a major whole tone. This results in a leading note that can be objectively tested, and is Reasonably close to the next note. It works well with the open 5th tuning of stringed instruments. This is sometimes called Pythagorean intonation. (A major whole tone followed by a minor whole tone will result in a harmonically tuned major 3rd.) You may exaggerate the inclination of the leading tone for expressive effect, particularly in Romantic music. Overuse of the raised leading tone, however, may lead to playing sharp, especially in the higher registers – an all too common problem among violinists.

An approach to intonation championed by Casals and common among the great soloists of the mid- to late 20th century, such as Heifetz, was called expressive intonation. This is characterised by exaggerated leading notes and slightly high pitch relative to the accompaniment. This approach results in wide major and narrow minor intervals, and it requires constant vibrato. When superbly done, it allows a soloist to cut through an orchestra like a knife. However, in an ensemble this approach can sound like cats fighting – everybody is a soloist and there's no blend. For the best and worst of this approach, listen to Heifetz performing, say, the Sibelius concerto; it takes your breath away. Then listen to Heifetz performing solo Bach; it grates like chalk on a blackboard.

Dorothy DeLay was a proponent of playing double-stops and ensemble chords

with harmonic intonation, and sequential passages with Pythagorean intonation, striking a balance between the requirements of harmonic and melodic intonation – as can be heard in the playing of Itzhak Perlman.

THE HARMONIC SERIES

On a stringed instrument, the sounding of a note results from a vibrating string set in motion by offsetting the string and letting go so that it rebounds (repeatedly when bowed). This creates waves in the string. Think of taking a rope, fixed at one end, and holding it and flicking it up and down at the other.

You can create any (whole) number of waves in a string, and, just as in water, waves can superimpose on other waves. An instrument string vibrates as a whole

(called the fundamental, or first harmonic), and in whole-number multiples (called overtones, or harmonics). This sequence is known as the harmonic series, and is heard as a pitch (the fundamental) with a certain tone colour, depending on the relative strengths of its harmonics.

You can hear the higher harmonics by preventing the motion of lower harmonics. For example, placing the finger lightly in the middle of a string blocks it from vibrating in one piece (the fundamental) but allows all the other harmonics, from the second one upwards, to vibrate, resulting in a pitch an octave higher than the open string. In a real string, with some stiffness and irregularity, the harmonics are not necessarily perfect whole-number multiples of the fundamental; they tend to be a bit flat. ▶

FIGURE 3 Sine wave (measured on an oscilloscope) showing the amplitude relative to time of a pure tone generated electronically

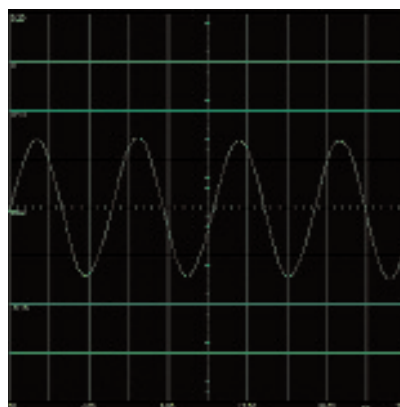


FIGURE 4 Amplitude relative to time of the same pitch as in figure 3 but this time played on the open string of a violin

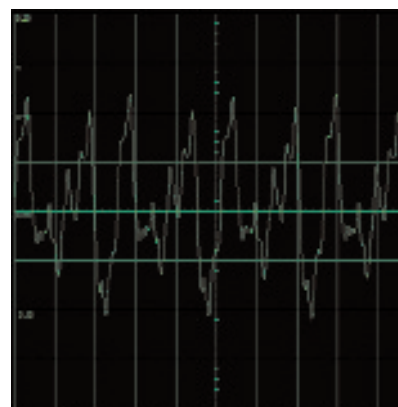


FIGURE 5 Frequency and amplitude of the same violin open string as in figure 4. Each peak represents a harmonic (the clutter at the bottom of the graph is noise)

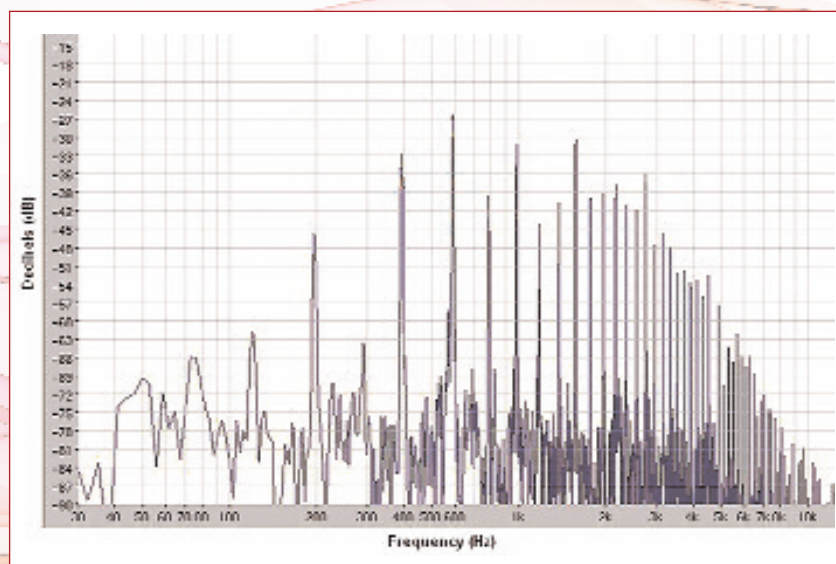
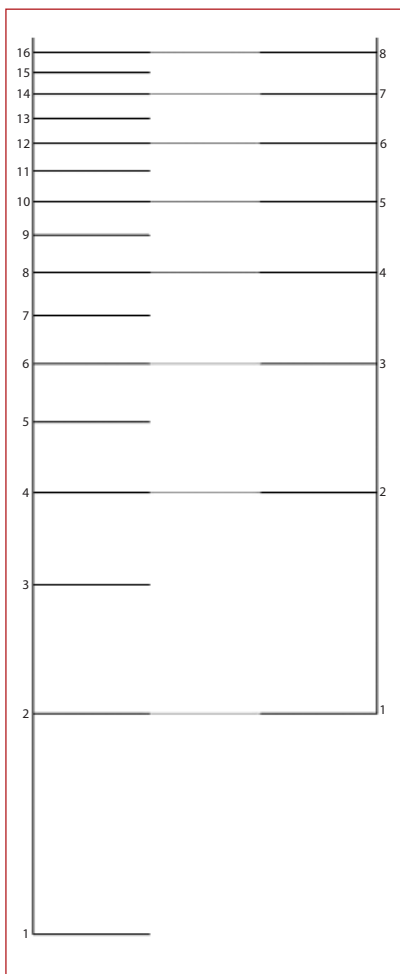


FIGURE 6 The coincidence of harmonics in an octave. The line on the left represents the lower note; the line on the right represents the note an octave higher



Each harmonic can be thought of as a pure tone of a frequency that is a whole-number multiple of the first harmonic (fundamental). **Figure 3** (page 57) shows the amplitude relative to time of a pure tone; the horizontal axis shows time, the vertical axis shows amplitude. This pure-tone

wave is called a sine wave because its amplitude is proportional to the sine of its duration.

Figure 4 (page 57) shows the same pitch played on an open string of a violin. You can see the periodicity of the sine tone. The zigzags are overtones – superimposed waves.

Another way to look at the same open string is in terms of frequency (rather than time) and amplitude, as shown in **figure 5** (page 57). Each peak illustrates a harmonic. It is interesting to note that on my violin the second harmonic is stronger than the fundamental.

The bow suppresses overtones by restricting string motion, depending on its placement. Placement nearer to the bridge allows higher multiples of the string to vibrate, resulting in a brighter tone; if the bow is placed further from the bridge, more harmonics are suppressed, resulting in a more muted tone.

MAJOR, MINOR AND PERFECT INTERVALS

Intervals are heard as consonant because when they are in tune some members of their harmonic series coincide. When they are out of tune, the nearly coincidental overtones beat, making that fact clearly audible. The more consonant the interval, the more shared overtones, and thus the more obvious it is when it is out of tune. So in a sense, consonance is defined by poor intonation.

For example, an octave has a frequency ratio of 2:1. The second harmonic of the lower note coincides with the first harmonic of the higher note; the sixth of the lower with the third of the upper, and so on (see **figure 6**). **Figure 7** shows this relationship for all the consonant intervals.

You can see from **figure 7** that as we move from perfect consonant intervals (octaves, 4ths and 5ths) to imperfect ones, the coincidence of harmonics occurs higher and higher in their respective harmonic series, reducing the strength of their relationships. This is why intonation for perfect intervals must be much more precise than that for imperfect ones. This more attenuated relationship allows more give in the playing of 3rds and 6ths.

TARTINI TONES

A Tartini tone is the lower, third tone heard when two consonant notes are played with firm bow control and precisely in tune. Tartini tones are very helpful when tuning consonant intervals, and they add richness to their tone.

Tartini tones are generated only in the ear, as demonstrated by the figures on page 60. **Figure 8** shows two sine tones, a minor 3rd apart (F sharp plus open A string). These notes were generated electronically and measured by an oscilloscope. The two notes are shown by the two tall purple peaks at their respective frequencies.

These two notes played together produce another, clearly audible, note (D, an octave below open D) that does not show up in the measurement, shown by the grey peak in **figure 8**. This is called a Tartini (or difference) tone.

When two consonant notes are played in tune, the overtone series from each of these notes' overtones coincide with members of the overtone series of their Tartini tone. When we hear part of an overtone series, our hearing system supplies the fundamental, even if it is not physically there. (That is why you can hear bass notes from loudspeakers that are too >

FIGURE 7 The coincidence of harmonics in all the consonant intervals: (from left) octave, 5th, 4th, major 3rd, minor 6th, minor 3rd and major 6th

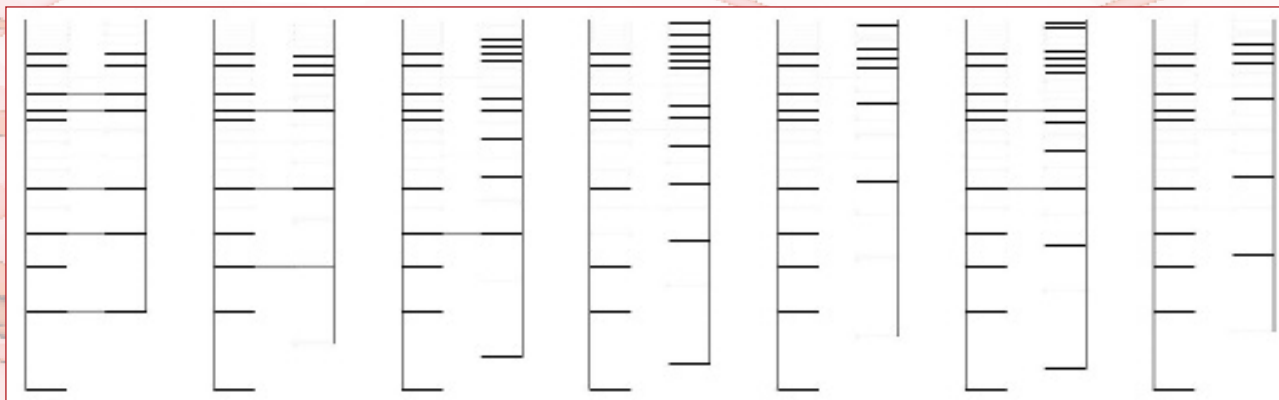
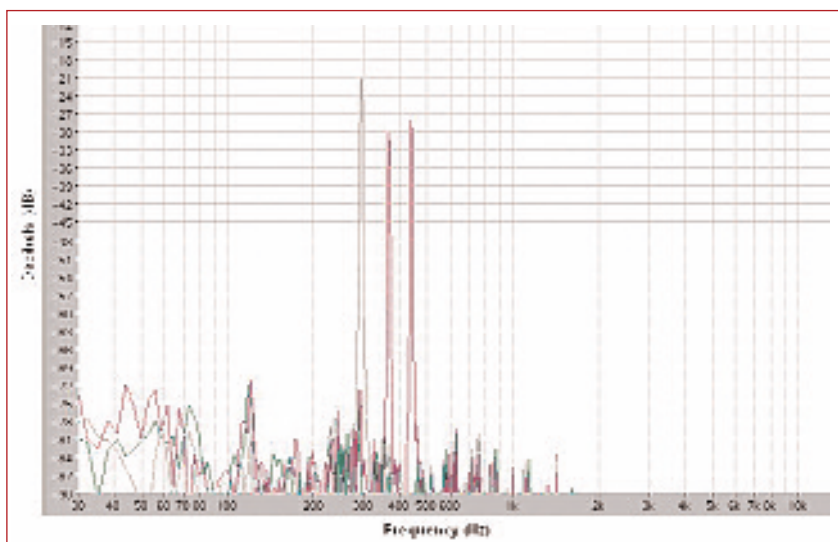


FIGURE 8 The purple peaks represent a minor 3rd (F sharp and A at open A string pitch) generated electronically and measured by an oscilloscope. The pitch of the Tartini tone heard when those two notes are played simultaneously is superimposed in grey. The Tartini tone is represented by the highest peak



small physically to reproduce those notes.) This reinforces perception of the Tartini tones that are generated in the inner ear

WHY PERFECT INTONATION IS IMPOSSIBLE

Perfect intonation is mathematically impossible because you can go from one note to another via a different series of perfect intervals and end up with slightly different frequencies.

For example, to reach a major 3rd above an open C string by 5ths, take the frequency ratio of the 5th, which is 3:2, and multiply it thus: $3/2 \times 3/2 \times 3/2 \times 3/2$ (or $(3/2)^4$); then divide that by 2×2 in

order to drop down two octaves.

The resulting frequency ratio is 1.265625. This is considerably higher than the frequency ratio for a pure major 3rd: 1.25 (5:4). This is why cellists often raise their C string slightly when playing in quartets. So what is going on here?

According to the Unique Prime Factor Theorem, every whole number has a unique set of prime factors. For instance, the number 14 is 2×7 .

Ignoring octave displacement for the sake of simplicity, all intervals can be considered multiplication by whole numbers. So if you go from one note to another via one set of intervals (multiply

by one set of whole numbers), you will not necessarily reach the same exact frequency via another set of intervals.

TIPS FOR BETTER INTONATION

Finally, here is some advice on how to work on your intonation in practical terms.

Get comfortable

It is important for you to get physically comfortable with your instrument. You may have perfectly good ears, but play out of tune because of a bad physical relationship with your instrument. Tension resulting from poor practice can result in inaccurate shifts, and even in an inability to hear yourself objectively when playing.

Record yourself

This can be painful, but also enlightening. It helps to record yourself with decent equipment so that you don't sound bad merely because of the quality of the recording itself. For violin tone, stereo makes a big difference.

Practise slowly

It takes time to hear pitch precisely, so practising fast too early while learning a piece will train fingers to fall imprecisely, and will dull objective listening.

Delay vibrato

Never use vibrato until you have a clear idea of pitch. While learning a piece try playing it entirely without vibrato, and then with a very small, centred vibrato, until the pitch relationships jell.

Practise scales and arpeggios accompanied by a drone

There is a simple note-generating program on the internet that you can use for this purpose:

www.nch.com.au/tonegen/index.html

Play duets

Intonation is the relationship among notes, and what better way to work on this relationship than in duets? The first violinist of the Biava Quartet, Austin Hartman, says that the quartet practises Bach chorales very slowly and without vibrato for intonation.

Play Bach

In addition to all their other virtues, Bach's sonatas and partitas for violin and his cello suites are excellent intonation studies. Every issue of melodic versus harmonic intonation relative to the open strings arises, forcing the player to decide consciously on a philosophy of intonation. ■

BEATS

Beats are the pulsing of two notes, and we use them to tune open strings. In tuning open 5ths, we match the third harmonic of the lower string to the second of the upper and tune until the beats slow down and disappear.

When two pure tones are played simultaneously they alternately reinforce and cancel each other out at the rate of the difference between their frequencies. As they approach each other in frequency, beating slows. The figure here shows an oscilloscope image of two sine tones slightly out of tune, beating.

